

Abstract

The subject of this dissertation is the mathematical treatise *The Book of the number* of Elijah Mizrahi, written in Constantinople at the end of the 15th century or the beginning of the 16th century.

Elijah Mizrahi

Elijah Mizrahi (c. 1450-1526), "Ha Re'em", was born in Constantinople. He was of Romaniote origin (his family was from Ottoman Greece and not from the Spanish exile). The Jewish community in Constantinople was one of the largest and most important in those days, especially after the expulsion of the Jews from Spain (in 1492). Mizrahi became a prominent personality in the Jewish community in Constantinople and occupied many public positions.

He was considered both by his contemporaries and by later generations the most important rabbinical religious authority in Constantinople and throughout the whole Ottoman Empire in his time. He wrote several treatises on religious subjects like his supercommentary on Rashi's commentary on the Tora. Mizrahi was also interested in scientific subjects. Besides *The Book of the number* he wrote a commentary on Ptolomey's *Almagest*¹.

The intellectual atmosphere in Constantinople during his time was rich and diverse.

One of his teachers in secular subjects was Mordechai Comtino, whose treatise *On Reckoning and Measurement* was one of the sources for his *Book of the Number*. One of his colleagues was the Karaite Calev Afendopolo, author of a commentary on the

¹St. Petersburg - Inst. of Oriental Studies of the Russian Academy C 128

Hebrew translation of the *Arithmetic* of Nicomachus made from Arabic by Kalonymus ben Kalonymus in 1317.

The Book of the number

Mizrahi's *Book of the number* (also known as *The Art of Number*) was widely used during his time and later, but did not receive in-depth research until now.

This great essay, approximately 200 pages, consists of three articles (chapters) dealing with arithmetic operations on integer numbers, fractions, and sexagesimal fractions (sexagesimal fractions were used in astronomic calculations). It also includes a chapter on square roots and cubic, proportions (numerical proportions, geometric proportions and musical proportions) and a chapter of 99 questions dealing with numbers and geometric subjects.

Mizrahi organizes each of his subjects following the same schema:

- First he defines and explains the arithmetic operation and the related notions.
- Second he describes the algorithms and gives examples. He also gives special attention to checking techniques of the arithmetic operation.
- He explains and proves the algorithms in a separate section at the end of the relevant chapter.

This structure of presenting first the algorithms (accompanied by examples) and the explanations and proofs in later chapters was standard since al-Khwarizmi, probably because it facilitated usage. If someone is interested in methods of computation and of solving problems he can find them in Mizrahi's book without explanations that are not indispensable, but if he wants further explanations he will find them at the end of the chapter.

This structure of the book certainly facilitated the publishing of the abridged edition of the book during the 16th century– *Compendium of Arithmetic*. This abridged edition contained only the algorithms presented in the first two chapters, and omitted the long explanations and proofs. It was also translated in Latin (partially) and printed in 1546. It was printed again in the beginning of the 19th century - 1809.

The thesis focuses on the first part (sha'ar) of the first chapter (article) of the book which includes the four basic arithmetic operations on integer numbers, written in the decimal position system.

The decimal position notation for numbers, which originated in India and was used by the Arabs already in 9th century, arrived in Europe at the beginning of the 13th century. Ibn Ezra used the positional notation in his *Book of Number*, but chose to use the first nine Hebrew letters and the zero sign instead of the Hindu-Arabic numerals. Fibonacci introduced the Hindu-Arabic numerals and the decimal position notation for numbers in his book *Liber Abaci* in 1202. This notation was known also in Byzantium at least from the middle of the 13th century, as testified by a manuscript dated 1252. Mizrahi uses both the Hebrew letters and the Hindu-Arabic numerals.

During the 13th-16th centuries many treatises dealing with arithmetic operations with Hindu-Arabic numerals were published in Europe. Most treatises were teaching arithmetic algorithms for the new positional system without providing theoretical arguments to justify them.

Elijah Mizrahi's book is different in this respect. Mizrahi systematically presents a large variety of algorithms; he discusses their advantages and also their disadvantages and carefully explains and proves them. Apparently he consulted several mathematical books.

Mizrahi based his work on many sources. He quotes Ibn Ezra, Euclid's *Elements* and Nicomachus' *Arithmetic*, but it would appear that he read also other books, most of them in Hebrew but some of them, perhaps, in other languages (Greek or Arabic). It is obvious that he used Comtino's treatise *On Reckoning and Measurement* and also another arithmetic essay by Isaac ben Moshe Ali named *The Art of Number*. Isaac ben Moses Ali is known only by this arithmetic essay that is extant in six manuscripts. In two of them there are marginalia signed Elijah Mizrahi. Some of these marginalia are very close to Mizrahi's text so we can suppose that Mizrahi used this treatise in his work.

The thesis' scheme

This thesis contains:

- The critical edition of the text (the first part of the first article) relying on 7 extant manuscripts (some of them complete) and the first printed edition from 1534. (This printed edition was issued in Constantinople by Mizrahi's son Israel after Mizrahi's death.)
- A thorough examination of the various arithmetic algorithms presented.
- A comparison of Mizrahi's arithmetical notions and algorithms with those used in other known treatises written by Hebrew mathematicians before Mizrahi's time. Mizrahi's work was compared also with arithmetic essays written about that time or earlier in other languages: Arab authors like al-Khwarizmi and al-Uqlidisi, Italian ones like Fibonacci's *Liber Abaci* and the *Treviso Arithmetic* and Byzantine authors like Maximis Planudes.
- A dictionary of mathematical notions found in Mizrahi's text and a comparison table with mathematical notions in earlier Hebrew texts.

- A list of remarks found in the text which reflect Mizrahi's views concerning different didactic issues, and a comparison with similar remarks in other Hebrew authors. Most of these remarks seem to be relevant today.

Conclusions

My aim was to describe the development of various arithmetic operations' algorithms beginning with Al-Khwarizmi. I also tried to determine whether the work of Mizrahi's is only a compilation of known materials or whether it includes some original innovations.

Indeed, one can point out some unique contributions of Mizrahi in several issues:

- Most algorithms are described to the minute details and are accompanied by rigorous explanations. Some explanations include algebraic considerations, although verbal.
- Mizrahi presents some unique arithmetic's algorithms, especially for multiplication and division.
- Mizrahi uses unique terms like "quality" and "quantity" to describe the role of Hindu-Arabic numerals in the positional notation.
- In the chapter dealing with sums of numbers' sequences Mizrahi presents unique methods for calculating certain types of sums and original explanations which we do not find in other mathematical treatises.
- Some of Mizrahi's explanations in the chapter dealing with sequences can be identified as pre-inductive methods. Such methods have not yet been found in Jewish authors except Gersonides.

The study of the first part of Mizrahi's book reveals a pedant scholar, who was well versed in the extensive arithmetic literature of his time, and who composed a comprehensive book summarizing the arithmetic knowledge that has been accumulated over 300 years, since the decimal positional system for writing numbers was introduced into Europe. The study of the other parts of the book can give a more complete picture of Elijah Mizrahi's achievements and I hope to pursue this research in the future.